**MMAT5010 Linear Analysis (2024-25): Homework 2**
**Deadline: 8 Feb 2025**

# Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard.

1. Recall that a subset $D$ of a normed space $X$ is said to be dense in $X$ if $\overline{D} = X$.

   (a) Show that if the finite sequence space $c_{00}$ is endowed with the $\|\cdot\|_1$-norm, then it is a dense subspace of $\ell_1$.

   (b) When $c_{00}$ is endowed with the $\|\cdot\|_\infty$, show that it is not a dense subspace of $\ell_\infty$.

2. (a) Show that every convergent sequence in a norm space is bounded, that is, $\sup_n \|x_n\| < \infty$ when $(x_n)$ is a convergent sequence in a normed space $X$.

   (b) For each pair of elements $x$ and $y$ in $\ell_\infty$, we define the product $x * y \in \ell_\infty$ by

   $$x * y(k) := x(k)y(k) \quad \text{for } k = 1, 2, \ldots.$$

   Show that the product $* : \ell_\infty \times \ell_\infty \to \ell_\infty$ is continuous, that is, $x_n * y_n \to x * y$ whenever $x_n, y_n \in \ell_\infty$ with $x_n \to x$ and $y_n \to y$.
   (Hint: use Part $(a)$)

$* * *$ **End** $* * *$